The Predictive Power of the Interest Rate for Industry-level TFP: A State-space Approach

Munechika Katayama*

Louisiana State University

Abstract

This paper analyzes the predictive power of the interest rate for various industry-level measures of productivity growth. Although industry-level data are only available at an annual frequency, by using the state space model and the Kalman filter, it is possible to perform the Granger-causality test at a quarterly frequency. The results highlight the heterogeneous nature of predictive power and suggest that the nonexogeneity of the Solow residual reported by Evans (1992) is due to manufacturing industries. In addition, two case studies on industries in which we can obtain an appropriate measure of capital utilization rate show that the forecasting ability of the interest rate diminishes after taking account of variable capital utilization in TFP growth.

1 Introduction

Although productivity shocks play an important role in modern macroeconomic models, we have limited knowledge about their usual empirical measures, the Solow residual. Although the Solow residual plays an important role in the RBC literature as an exogenous driving force, Evans (1992) casts doubt on its exogeneity by showing that the observed Solow residual is Granger-caused by money, interest rates, and government spending.

There have been many studies that try to understand the nature of the observed Solow residual and they have offered several possibilities why the conventionally measured Solow residual might fail to be the true measure of technological progress, such as variable factor utilization, variable markup rate, increasing returns to scale, and sectoral reallocation.

*I thank my advisor Professor James Hamilton for his encouragements and helpful comments.

Partly because of data availability, however, most studies are based on aggregate data. We argue that we also need to understand the nature of technological progress at the disaggregated level. It is true that the aggregate one-sector model is handy to analyze, but at the same time, our economy consists of heterogeneous industries. So we will obtain better insights from more disaggregated level of analysis. For instance, it is possible that some industries are better characterized by increasing returns to scale, but other industries are not.

There are several studies analyzing industry-level productivity. Many of them focus on manufacturing industries, such as Basu and Fernald (1995), Basu (1996), Basu and Kimball (1997), and Burnside, Eichenbaum, and Rebelo (1995, 1996), among others. A few exceptions are Basu and Fernald (2001), Basu, Fernald, and Shapiro (2001), and Basu, Fernald, and Kimball (2004), which cover the entire industries in the economy and try to establish the link between the industry-level productivity and the aggregate productivity. Most of the above studies take account of possibilities that contaminate the observed measure of technological progress. However, the fundamental question raised by Evans (1992) still remains unanswered at the disaggregated level.

This paper focuses on the following question: What are the sources of the Granger-causality found in the aggregate Solow residual? More specifically, we decompose the aggregate Solow residual into industry-level TFP growth and investigate sources of the Granger-causality within highly aggregated groups of industries and more disaggregated industries systematically.

Although Evans (1992) uses quarterly data, industry-level productivity series are only available at an annual frequency. It is often the case that macroeconomic variables of interest are available at different frequencies. In such circumstances, we usually need either to convert high frequency variables into lower frequency or to interpolate low frequency variables with appropriate signals. Both approaches have some problems. If we convert high frequency data into a lower frequency, we may loose some important information. On the other hand, there is no guarantee that the variable used to interpolate low frequency series is a good proxy for the true time series behavior and it is likely to create an errors-in-variable problem.

In this paper, we investigate an alternative way to handle observations at different frequencies. Our approach is based on the state space model. Together with the Kalman filter, the state space model provides a highly flexible framework that can deal with unobserved component in the estimation model. For example, Hamilton (1985) and Burmeister, Wall, and Hamilton (1986) use the state space model to uncover the market's expected rate of inflation based on observed interest rates and inflation.

Recently it has also been popular to use the state-space representation and the Kalman filter to take DSGE models to the data. In order to cope with the stochastic singularity problem in DSGE models, people include a variety of additional structural disturbances so that the number of shocks equals to the number of observables in the estimation, or add measurement errors.[1] In such situations, regardless of whether one uses maximum likelihood estimation or a Bayesian approach, it is necessary to rely on the Kalman filter. There are many other advantages of the state space model, for example, its capabilities of handling missing observations, temporal and contemporaneous aggregation, and data irregularities such as those arising from data revisions. However, most applications of the state space model have been limited to simple introduction of unobserved components as described above.

It is not new to use the state space model in order to cope with missing observations (see, for example, Harvey and Pierse, 1984). However, such applications mainly focus on uncovering missing observations in time series. By combining missing observations and temporal aggregation, we will be able to analyze time series data at different frequencies in one-step without converting data frequencies beforehand. In such a unified framework, we can perform statistical tests based on estimates, which are entirely consistent with observed realizations even at different frequencies. Not only do we decompose the aggregate Solow residual, but also we perform Granger-causality test at a quarterly frequency by using the

[1]See, for example, Smets and Wouters (2003), Ireland (2004), and references therein.

state space model in order to explore the source of the Granger-causality.

The rest of the paper is organized as follows. Section 2 describes how to measure the industry-level TFP based on gross outputs and explains how we incorporate annual data into the state space model with a quarterly frequency. Section 3 updates findings in Evans (1992) and analyzes the Granger-causality at the aggregate level using the disaggregated data. Section 4 extends our framework to control for variable capital utilization. Finally Section 5 concludes.

2 Method

In this section, we describe our procedure. Section 2.1 explains how to measure the industry-level productivity and establish the link between the industry-level productivity and the aggregate productivity measures. Details about the state space model and the Kalman filter are given in Section 2.2. Description of the data is in Section 2.3.

2.1 Measuring the Industry-level Productivity Growth

When we analyze the relation between the aggregate economy and underlying industry activities, we will encounter a dilemma. Although GDP is value added, an appropriate measure of output at the industry or firm level is gross output if we are interested in the industry-level productivity. Value added is defined as the difference between the value of gross output and the cost of intermediate inputs. Goods produced by industries or firms are either consumed as final products or used as intermediate inputs for other production processes. To avoid double counting, an aggregate measure of output is value added, which is net of payments to intermediate inputs. If we use aggregate value added, all flow of intermediate inputs will be canceled out and hence it does not cause any problem of ignoring intermediate inputs. However, interindustry flows of intermediate goods do not cancel out even at the one-digit industry level.

Not only does value added have no physical counterparts in terms of tangible goods and services as industry-level outputs, but also we need an additional restrictive assumption that the gross-output production function takes the following form:[2]

$$Q_i = F\left(A_i G\left(K_i, L_i\right), E_i, M_i\right). \tag{1}$$

where Q_i is gross output in industry i, $G(\cdot)$ is a value-added function, K_i and L_i are primary inputs of capital and labor, A_i is a technology index, E_i energy usage in production process, and M_i is material inputs. For notational simplicity, time-subscripts are suppressed. This separability condition requires that the marginal rate of substitution between capital and labor is independent of energy and material inputs and implies that technological innovation only improves the productivity of capital and labor, but not intermediate inputs. A more general specification would be

$$Q_i = Z_i F\left(K_i, L_i, E_i, M_i\right). \tag{2}$$

Once difference between (1) and (2) is that Z_i is gross-output-augmenting technology, while A_i is value-added-augmenting technology. In order to distinguish A_i and Z_i, hereafter, we call $\frac{dA}{A}$ the Solow residual and TFP (or KLEM TFP) refers to $\frac{dZ}{Z}$.

While we want to establish the link between the aggregate Solow residual and the industry-level TFP, simple aggregation of the industry-level TFP does not work because there exist conceptual differences as described above. The dilemma is coming from the fact that appropriate measures of outputs and specifications of the production function are different.

To overcome the dilemma, we will take the following strategy: (i) Calculate the industry-level technological progress; (ii) Aggregate over industries to construct the aggregate measure of technological progress based on gross output; (iii) Using an accounting identity, convert the gross-output based measure of productivity into the one based on the value-

[2]This weak separability condition has been well-known in the production function literature. Sato (1975, Ch.2) provides a comprehensive summary. In the context of productivity analysis, see Jorgenson (1990) and Hulten (2001).

added function concept.[3]

Under perfect competition and constant returns to scale, a standard growth accounting approach based on (2) gives us the industry-level TFP

$$\frac{dZ_i}{Z_i} = \frac{dQ_i}{Q_i} - \left\{ c_i^K \frac{dK_i}{K_i} + c_i^L \frac{dL_i}{L_i} + c_i^E \frac{dE_i}{E_i} + c_i^M \frac{dM_i}{M_i} \right\}, \tag{3}$$

where $c^X = \frac{P_i^X X_i}{P_i^Q Q_i}$ for any variable X_i and P^X denotes a price of any good X_i.

Aggregation over industries can be done by defining the Divisia index of aggregate KLEM TFP as follows:

$$\frac{dZ}{Z} = \sum_{i=1}^{N} \omega_i^Q \frac{dZ_i}{Z_i}, \tag{4}$$

where $\omega_i^Q = \frac{P_i^Q Q_i}{\sum_{i=1}^N P_i^Q Q_i}$. Notice that (4) holds at any level of aggregation, regardless of whether we are aggregating over the entire economy or a subset of the economy.

In order to convert the aggregate measure of KLEM TFP into the aggregate Solow residual, we will start from an accounting identity. Let Y be aggregate real value added. Since nominal value added is the difference between the value of gross output and the expenditure for energy and material inputs,

$$P^Y Y = P^Q Q - P^E E - P^M M,$$

where variables without subscripts are aggregate measures. Following Sato (1976), we define the Divisia quantity index for aggregate real value added

$$\frac{dY}{Y} = \frac{P^Q Q}{P^Y Y}\frac{dQ}{Q} - \frac{P^E E}{P^Y Y}\frac{dE}{E} - \frac{P^M M}{P^Y Y}\frac{dM}{M} = (1 + s^E + s^M)\frac{dQ}{Q} - s^E \frac{dE}{E} - s^M \frac{dM}{M}, \tag{5}$$

where $s^X = \frac{P^X X}{P^Y Y}$ for any variable X.

Using $\frac{dQ}{Q} = \frac{dZ}{Z} + c^K \frac{dK}{K} + c^L \frac{dL}{L} + c^E \frac{dE}{E} + c^M \frac{dM}{M}$ and $(1 + s^E + s^M)c^X = \frac{P^Q Q}{P^Y Y}\frac{P^X X}{P^Q Q} =$

[3]This strategy is similar to the one used in Basu and Fernald (2001), Basu, Fernald, and Shapiro (2001), and Basu, Fernald, and Kimball (2004).

$\frac{P^X X}{P^Y Y} = s^X$ for any X, we can rewrite (5) to obtain

$$\frac{dY}{Y} = s^K \frac{dK}{K} + s^L \frac{dL}{L} + s^Q \frac{dZ}{Z}.$$

Since the Solow residual $\frac{dA}{A}$ is computed as $\frac{dA}{A} = \frac{dY}{Y} - s^K \frac{dK}{K} - s^L \frac{dL}{L}$, we obtain the relation between the aggregate Solow residual and the industry-level KLEM TFP as

$$\frac{dA}{A} = s^Q \frac{dZ}{Z}.$$

Since the value of gross output is greater than the value of value added by construction, this tells us that the value-added based measure of productivity is greater in magnitude than the TFP based on all inputs. Using (4), we can achieve the relation between the aggregate Solow residual and the industry-level TFPs:

$$\frac{dA}{A} = \sum_{i=1}^{N} d_i \frac{dZ_i}{Z_i}$$

where $d_i = \frac{P_i^Q Q_i}{\sum_{i=1}^N P_i^Y Y_i}$ is called the Domar weight (Domar, 1961).

Notice that any change in assumptions, such as imperfect competition, increasing returns to scale, or variable factor utilization, will alter the form of (3). However, the remaining steps are unchanged.

So far, technological progress has been characterized as a continuous-time growth rate (i.e., the Divisia index). In our empirical study, we employ a commonly used discrete approximation to the Divisia index, namely the Törnqvist index. The Törnqvist index is defined as the differences in logs and the weights are equal to the average of current and previous periods.

2.2 State space Model

A state space model consists of two sets of equations:

$$\text{State equation} \quad \boldsymbol{\xi}_{t+1} = \boldsymbol{\Phi}\boldsymbol{\xi}_t + \boldsymbol{\varepsilon}_{t+1} \tag{6}$$

$$\text{Observation equation} \quad \boldsymbol{y}_t = \boldsymbol{\Psi}_t\boldsymbol{\xi}_t + \boldsymbol{b}_t + \boldsymbol{\nu}_t \tag{7}$$

where $\boldsymbol{\xi}_t$ is the state vector containing unobserved variables and $\boldsymbol{y}_t$ is a vector of observables. $\boldsymbol{\Phi}$, $\boldsymbol{\Psi}_t$, and $\boldsymbol{b}_t$ are matrices of parameters. The $\boldsymbol{\varepsilon}_{t+1}$ and $\boldsymbol{\nu}_{t+1}$ are vectors of white noise:

$$E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_s'] = \begin{cases} \boldsymbol{Q} & \text{for } t = s \\ \boldsymbol{0} & \text{otherwise} \end{cases} \qquad E[\boldsymbol{\nu}_t\boldsymbol{\nu}_s'] = \begin{cases} \boldsymbol{R} & \text{for } t = s \\ \boldsymbol{0} & \text{otherwise} \end{cases}$$

$$E[\boldsymbol{\varepsilon}_t\boldsymbol{\nu}_s'] = \boldsymbol{0} \quad \text{for all } t \text{ and } s.$$

The state equation describes the dynamics of unobserved variables and the observation equation relates observed data with unobserved variables contained in the state vector.

To carry out an empirical estimation, we need to make use of the Kalman filter due to the existence of unobservables. The Kalman filter is an algorithm to recursively estimate the state vector at time t given an initial condition and observations up to time t. Intuitively, the Kalman filter delivers the optimal estimator of unobserved components as each new piece of information becomes available.[4] When all observations have been processed, it yields the optimal estimator of the state vector that is consistent with observed realizations, given a model of interactions between observed and unobserved variables. This estimator contains all the information necessary to make optimal predictions of future values of the state and the observations.

Given parameter values and an initial condition, the Kalman filter gives us the sequence

[4] Derivation of the Kalman filter is well described in Hamilton (1994, Ch.13). Key equations are reproduced in Appendix.

of optimal estimates of the state vector, $\{\hat{\boldsymbol{\xi}}_{t+1|t}\}_{t=1}^{T}$, and the associated sequence of MSEs, $\{\boldsymbol{P}_{t+1|t}\}_{t=1}^{T}$, where

$$\begin{aligned}
\hat{\boldsymbol{\xi}}_{t+1|t} &= E\left[\boldsymbol{\xi}_{t+1}|\Omega_t\right], \\
\boldsymbol{P}_{t+1|t} &= E\left[\left(\xi_{t+1|t} - \hat{\boldsymbol{\xi}}_{t+1|t}\right)\left(\xi_{t+1|t} - \hat{\boldsymbol{\xi}}_{t+1|t}\right)'\right], \\
\Omega_t &\equiv [\ \boldsymbol{y}_1', \quad \boldsymbol{y}_2', \quad \cdots, \quad \boldsymbol{y}_t'\ ]'.
\end{aligned}$$

We will maximize the log-likelihood function and at each iteration of numerical maximization, the Kalman filter is invoked, and the log-likelihood function is evaluated based on $\{\hat{\boldsymbol{\xi}}_{t+1|t}\}_{t=1}^{T}$ and $\{\boldsymbol{P}_{t+1|t}\}_{t=1}^{T}$, given parameter values at each iteration.

If there are missing observation, we just need to either exclude those missing observations from a vector of observables (i.e., the dimension of $\boldsymbol{y}_t$ varies over time) or assign zeros for them. Even when we introduce missing observations in the Kalman filter, they do not affect the derivation and validity of the Kalman filter. The procedure of computing the gradient of the log-likelihood function and the information matrix remains unaffected as well.[5]

2.2.1 Specifications in the State Equation

Let us define

$$\boldsymbol{k}_t = [\ \Delta w_{1,t} \quad \cdots \quad \Delta w_{N,t} \quad \Delta \boldsymbol{x}_t'\ ]', \tag{8}$$

where $\Delta w_{i,t}$ is the unobserved quarterly TFP for the industry i at time t, and $\boldsymbol{x}_t$ is a vector of macroeconomic variables that will be specified below. As will be discussed later, it is necessary to include the current value and six lags of the unobserved technological growth at a quarterly frequency into the state vector $\boldsymbol{\xi}_t$. Then the state vector can be represented as

$$\boldsymbol{\xi}_{t+1} = [\ \boldsymbol{k}_{t+1}' \quad \boldsymbol{k}_t' \quad \cdots \quad \boldsymbol{k}_{t-5}' \quad 1\ ]'. \tag{9}$$

[5]See Harvey (1989) for more details. The procedure and formulas are presented in Appendix.

We could include a constant as an exogenous variable in the state equation. Instead, we include 1 in the last element of the state vector because it is more convenient in order to incorporate a constant term in the dynamics. We assume that $\boldsymbol{k}_{t+1}$ follows VAR(4):

$$\boldsymbol{k}_{t+1} = \boldsymbol{c} + \Phi_1 \boldsymbol{k}_t + \Phi_2 \boldsymbol{k}_{t-1} + \Phi_3 \boldsymbol{k}_{t-2} + \Phi_4 \boldsymbol{k}_{t-3} + \tilde{\boldsymbol{\varepsilon}}_{t+1}, \tag{10}$$

where

$$\boldsymbol{\Phi}_j = \begin{bmatrix} \rho_{11}^{(j)} & \cdots & \rho_{1N}^{(j)} & \boldsymbol{g}_1^{(j)\prime} \\ \vdots & \ddots & \vdots & \vdots \\ \rho_{N1}^{(j)} & \cdots & \rho_{NN}^{(j)} & \boldsymbol{g}_N^{(j)\prime} \\ \boldsymbol{h}_1^{(j)} & \cdots & \boldsymbol{h}_N^{(j)} & \boldsymbol{f}^{(j)} \end{bmatrix} \qquad \text{for } j = 1, \cdots, 4,$$

and $E[\tilde{\boldsymbol{\varepsilon}}_{t+1} \tilde{\boldsymbol{\varepsilon}}_{t+1}'] = \boldsymbol{Q}$ and $\boldsymbol{Q}$ is assumed to be diagonal.

We are interested in whether macroeconomic variables help predict the behavior of the unobserved quarterly technological progress. That is, we will test the null hypothesis that $\boldsymbol{g}_i^{(1)} = \cdots = \boldsymbol{g}_i^{(4)} = \boldsymbol{0}$ for the industry i.

The remaining components of the state equation are just identities, such as

$$\boldsymbol{k}_t = \boldsymbol{k}_t, \quad \cdots, \quad \boldsymbol{k}_{t-5} = \boldsymbol{k}_{t-5}, \quad 1 = 1. \tag{11}$$

So the corresponding parts of disturbances are set to be zero.

2.2.2 Specifications in the Observation Equation

Industry-level TFP In order to carry out the Granger-causality test for disaggregated industries at a quarterly frequency, we relate the observed annual industry-level TFP growth with the unobserved quarterly TFP growth rates. It is worthwhile to consider the relation between the annual technological growth and the quarterly technological growth because it will determine the specification of the observation equation.

There are at least two possible ways of measuring the annual technology level using quarterly series. One is to observe the annual technology level at a particular point in time (e.g., in the fourth quarter). The other one is to measure the annual technology level as the average of the quarterly technology levels.

We assume that annual observations of the technology level are available in the fourth quarter of each year. Let $\{w_{i,t}\}_{t=1}^{T}$ and $\left\{z_{i,t/4}\right\}_{t/4=1}^{T/4}$ be sequences of log of quarterly and annual technology levels in industry i, respectively. The time subscript t is a quarterly time index. If the annual technology level is measured, say, in the fourth quarter, we have $z_{i,t/4} = w_{i,t}$ and thus the annual TFP growth is expressed as $\Delta z_{i,t/4} \equiv \Delta w_{i,t}^{*} = w_{i,t} - w_{i,t-4}$. On the other hand, if we assume that the annual technology level is the average of quarterly technology levels, then

$$z_{i,t/4} = \frac{1}{4}\{w_{i,t} + w_{i,t-1} + w_{i,t-2} + w_{i,t-3}\}$$

and hence

$$\begin{aligned}\Delta z_{i,t/4} &= \frac{1}{4}\left\{\Delta w_{i,t}^{*} + \Delta w_{i,t-1}^{*} + \Delta w_{i,t-2}^{*} + \Delta w_{i,t-3}^{*}\right\} \\ &= \frac{1}{4}\{\Delta w_{i,t} + 2\Delta w_{i,t-1} + 3\Delta w_{i,t-2} + \\ &\qquad 4\Delta w_{i,t-3} + 3\Delta w_{i,t-4} + 2\Delta w_{i,t-5} + \Delta w_{i,t-6}\}. \end{aligned} \tag{12}$$

Obviously the former specification may be subject to various factors. For example, if there is the Christmas effect on the industry-level TFP, as reported in Barsky and Miron (1989), observing the technology level in the fourth quarter does not reflect true level of technology.

Notice that since (12) is an identity, we do not include either a constant term or an error term. Furthermore, since $\Delta z_{i,t/4}$ is the annual TFP growth observed in the fourth quarter, (12) holds only in the fourth quarter. The vector of observables $\boldsymbol{y}_t$ does not contain $\Delta z_{i,t/4}$ in other quarters, and hence the dimension of $\boldsymbol{y}_t$, $\boldsymbol{\Psi}_t$, $\boldsymbol{b}_t$, and $\boldsymbol{\nu}_t$ depends on time. Let N be the number of industries and let n and m denote the number of quarterly observable series and the size of the state vector. Then in the fourth quarter, $\boldsymbol{y}_t$, $\boldsymbol{b}_t$, and $\boldsymbol{\nu}_t$ are $((n+N) \times 1)$ vectors and $\boldsymbol{\Psi}_t$ is an $((n+N) \times m)$ matrix. In other quarters, the dimension of $\boldsymbol{y}_t$, $\boldsymbol{b}_t$, and

$\boldsymbol{\nu}_t$ is $(n \times 1)$ and $\boldsymbol{\Psi}_t$ becomes an $(n \times m)$ matrix.

Aggregate Solow residual As described above, the aggregate Solow residual is the weighted sum of the industry-level TFP with the Domar weight. It is likely that the conventionally derived aggregate Solow residual differs from the measure of aggregate technological progress based on the TFP at the industry level. It is because the conventionally derived aggregate Solow residual is based on the assumption that the aggregate value-added production function exists and there will be aggregation errors as well. We still believe that the conventionally derived Solow residual will give us some useful signals for unobserved quarterly TFP growth. Hence, we model the relation between the aggregate Solow residual and the industry-level TFPs by including a constant term and an aggregation error term as follows:

$$\Delta a_t = \sum_{i=1}^{N} d_i \Delta w_{i,t} + b + \nu_t, \tag{13}$$

where Δa_t is the growth rate of the conventionally derived Solow residual, and $d_i = \frac{P_i^Q Q_i}{P^Y Y}$ is the Domar weight.

Macroeconomic variables We observe macroeconomic variables every quarter. Thus, corresponding observations are nothing but identities.

$$\Delta \boldsymbol{x}_t = \Delta \boldsymbol{x}_t \tag{14}$$

This means that a part of components in the state vector is perfectly observable at a quarterly frequency.

2.2.3 Matrix Form

Equations (12)-(14) form our observation equation, $\boldsymbol{y}_t = \boldsymbol{\Psi}_t \boldsymbol{\xi}_t + \boldsymbol{b}_t + \boldsymbol{\nu}_t$. To summarize, in the matrix form:

$$
\boldsymbol{y}_t = \begin{cases} [\ \Delta a_t \quad \Delta \boldsymbol{x}_t' \ ]' & \text{if } t = \text{ Q1, Q2, or Q3} \\ [\ \Delta a_t \quad \Delta \boldsymbol{x}_t' \quad \Delta z_{1,t/4} \quad \cdots \quad \Delta z_{N,t/4} \ ]' & \text{if } t = \text{ Q4} \end{cases} \tag{15}
$$

$$
\boldsymbol{\Psi}_t = \begin{cases} \begin{bmatrix} \boldsymbol{\psi}_1' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & 0 \\ \boldsymbol{\Psi}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \end{bmatrix} & \text{if } t = \text{Q1, Q2, or Q3} \\ \begin{bmatrix} \boldsymbol{\psi}_1' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & 0 \\ \boldsymbol{\Psi}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \\ \frac{1}{4}\boldsymbol{\Psi}_3 & \frac{1}{2}\boldsymbol{\Psi}_3 & \frac{3}{4}\boldsymbol{\Psi}_3 & \boldsymbol{\Psi}_3 & \frac{3}{4}\boldsymbol{\Psi}_3 & \frac{1}{2}\boldsymbol{\Psi}_3 & \frac{1}{4}\boldsymbol{\Psi}_3 & 0 \end{bmatrix} & \text{if } t = \text{Q4} \end{cases} \tag{16}
$$

$$
\boldsymbol{b}_t = \begin{cases} [\ b \quad \mathbf{0}' \ ]' & \text{if } t = \text{ Q1, Q2, or Q3} \\ [\ b \quad \mathbf{0}' \quad 0 \quad \cdots \quad 0 \ ]' & \text{if } t = \text{ Q4} \end{cases} \tag{17}
$$

$$
\boldsymbol{\nu}_t = \begin{cases} [\ \nu_t \quad \mathbf{0} \ ]' & \text{if } t = \text{Q1, Q2, or Q3} \\ [\ \nu_t \quad \mathbf{0} \quad 0 \quad \cdots \quad 0 \ ]' & \text{if } t = \text{Q4} \end{cases} \tag{18}
$$

where

$$
\boldsymbol{\psi}_1' = [\ d_1 \quad \cdots \quad d_N \quad \mathbf{0} \ ] \tag{19}
$$

$$
\boldsymbol{\Psi}_2 = [\ \mathbf{0}' \quad \cdots \quad \mathbf{0}' \quad \boldsymbol{I} \ ] \tag{20}
$$

$$
\boldsymbol{\Psi}_3 = \begin{bmatrix} 1 & & 0 & 0 \\ & \ddots & & 0 \\ 0 & & 1 & 0 \end{bmatrix} \tag{21}
$$

$$E[\boldsymbol{\nu}_t\boldsymbol{\nu}_t'] = \boldsymbol{R}_t = \begin{cases} \begin{bmatrix} r & \mathbf{0}' \\ \mathbf{0} & \mathbf{0} \end{bmatrix} & \text{if } t = \text{Q1, Q2, or Q3} \\ \begin{bmatrix} r & \mathbf{0}' & \mathbf{0}' \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} & \text{if } t = \text{Q4} \end{cases} \tag{22}$$

For the state equation, equations (9)-(11) characterize the dynamic behavior of the state variables. In the matrix form, the state equation (6) can be expressed as:

$$\begin{bmatrix} \boldsymbol{k}_{t+1} \\ \boldsymbol{k}_t \\ \boldsymbol{k}_{t-1} \\ \boldsymbol{k}_{t-2} \\ \boldsymbol{k}_{t-3} \\ \boldsymbol{k}_{t-4} \\ \boldsymbol{k}_{t-5} \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 & \boldsymbol{\Phi}_3 & \boldsymbol{\Phi}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{c} \\ \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{k}_t \\ \boldsymbol{k}_{t-1} \\ \boldsymbol{k}_{t-2} \\ \boldsymbol{k}_{t-3} \\ \boldsymbol{k}_{t-4} \\ \boldsymbol{k}_{t-5} \\ \boldsymbol{k}_{t-6} \\ 1 \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{\varepsilon}}_{t+1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ 0 \end{bmatrix} \tag{23}$$

where

$$E[\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}_{t+1}'] = \begin{bmatrix} \boldsymbol{Q} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \mathbf{0} & & & \mathbf{0} & \mathbf{0} \\ \mathbf{0}' & \cdots & \cdots & \mathbf{0}' & 0 \end{bmatrix} \tag{24}$$

$\boldsymbol{Q}$ is assumed to be diagonal.

2.3 Data

For the industry-level data, we use the data set compiled by Dale Jorgenson and his colleagues. The data set consists of the industry-level production inputs and outputs as well as corresponding price indices.[6] It covers 35 industries of the U.S. private economy, which corresponds to one-digit non-manufacturing industries and two-digit manufacturing industries, from 1958 to 1996.

One advantage of Jorgenson's data set is that variables in the data set are constructed such that they are suitable for economic analysis. For example, labor inputs are constructed from the *Census of Population Survey*, the *Current Population Survey*, and the National Income and Product Account (NIPA). Using data from those primary sources, they construct labor inputs (hours worked) by taking account of different types of workers, such as sex, class of workers (employee, self-employed, or unpaid), age, and education. Capital inputs are also carefully constructed in a similar fashion, by classifying capital inputs into very detailed categories.[7]

Although there are a couple of alternative data sets, there are some shortcomings for our purpose. For example, although it has longer sample periods, the Multifactor Productivity database from the BLS does not cover the entire U.S. economy at the disaggregated level.[8] Another industry-level data set covering the entire economy is available from the BEA. The BEA data set contains gross output, intermediate inputs, nominal factor shares, and so on. Necessary series are available only from 1977, however. Furthermore, there is a discontinuity in terms of industry classification due to the revision in the SIC code. For these reasons, we will employ Jorgenson's data set.

For macroeconomic variables, all variables used here follow the description in Evans (1992).

[6] Available at http://post.economics.harvard.edu/faculty/jorgenson/data/35klem.html

[7] More detailed description of the data set can be found in Jorgenson, Gollap, and Fraumeni (1987), Jorgenson (1990), and Jorgenson and Stiroh (2000).

[8] Although NBER Industry Database can be another candidate, it covers only manufacturing industries as well.

Table 1: Granger-causality Test for 1960:I - 1996:IV

Variables	W	df	p-value
M1	7.799	4	0.099
T-Bill	18.474	4	0.001
Gov	2.236	4	0.693
Oil	1.195	4	0.879

Note: The second column of the table shows the Wald statistics for the null hypothesis of no Granger-causality.

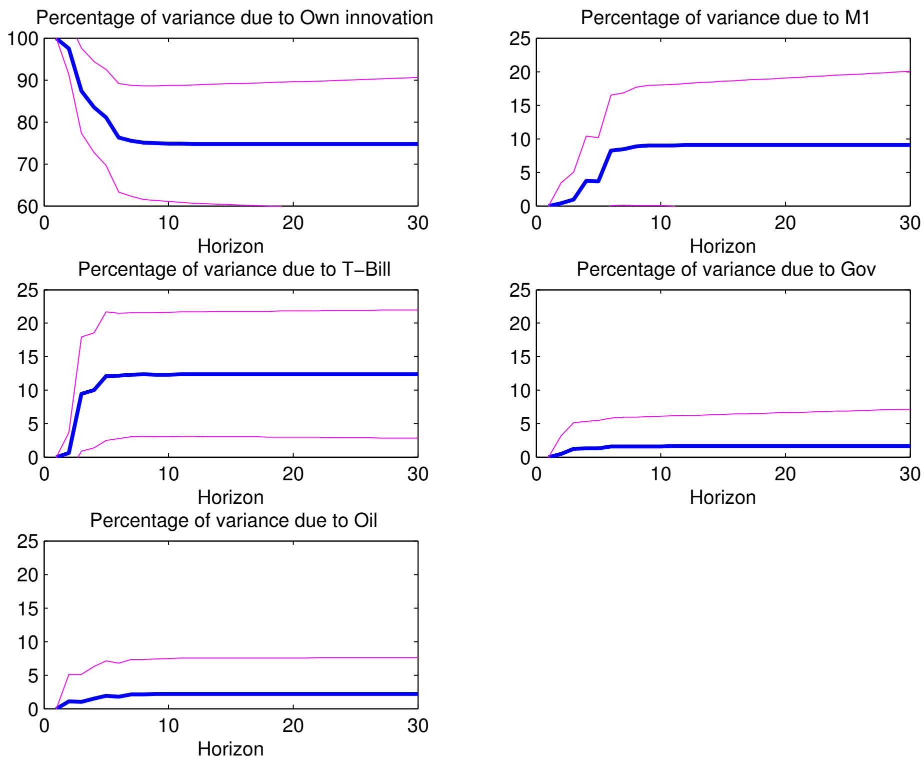

Figure 1: Decomposition of variance in the Solow residual

Following Evans (1992), the order of orthogonalization is as follows: Solow residual, M1, T-Bill, Oil, and Gov. Thick lines represent point estimates and thin lines are corresponding two-standard-error band computed by Monte Carlo with 1000 repetitions.

3 Empirical Results

We start off our empirical analysis by updating the Granger-causality test in Evans (1992). Table 1 reports the Granger-causality test results for the sample period 1960:I - 1996:IV. With an extended sample period, the predictive power of macroeconomic variables declines.

In his study, only oil price fails to Granger-cause the aggregate Solow residual. On the contrary, in this sample period, money and government expenditure no longer Granger-cause the Solow residual at the 5% level. The predictive power of the interest rate still remains statistically significant at the 1% level. Variance decomposition results are shown in Figure 1. While Evans (1992) reports that about 70% of the variance for 16-quarter-ahead forecasting error for the Solow residual can be attributable to its own innovations, in this extended sample, own innovations to the Solow residual account for about 75% of the variance. If we look at the percentage of forecasting variance explained by individual variables, the effect of the interest rate appears to be quantitatively significant as well.

For the reason described above, we will focus on the predictability of the interest rate on the aggregate Solow residual and try to see the source of the predictability at the disaggregated level. Thus, a vector of macroeconomc variables $\Delta \boldsymbol{x}_t$ in (8) becomes just a scalar, changes in 3-month T-Bill rates. Focusing on the interest rate also helps handle disaggregated data. As the level of disaggregation increases, it will be difficult to include all variables used in Evans (1992) since the number of parameters and the dimension of the state vector increases dramatically.

We will first decompose the aggregate Solow residual into two industries (i.e., $N = 2$), namely non-manufacturing and manufacturing. Figure 2 compares the impulse response function from the 2-industry model with the one from a bivariate VAR(5) of the aggregate Solow residual and interest rate. The lag length of VAR is chosen based on the LR criterion, AIC, and BIC. They are responses to one unit interest rate innovation. As you can see, the impulse response from the state space model successfully mimics the one obtained from VAR. This suggests that our framework that deals with missing observations and contemporaneous and temporal aggregation using the state space model may help identify the source of the interest rate predictive power.[9]

The upper panel of Table 2 shows the maximum log-likelihood values and the likelihood ratio test results for the 2-industry model. The null hypothesis is that $g_i^{(1)} = \cdots = g_i^{(4)} = 0$,

[9] Also we conduct a simple Monte Carlo exercise to show the validity of our approach. The results are presented in Appendix.

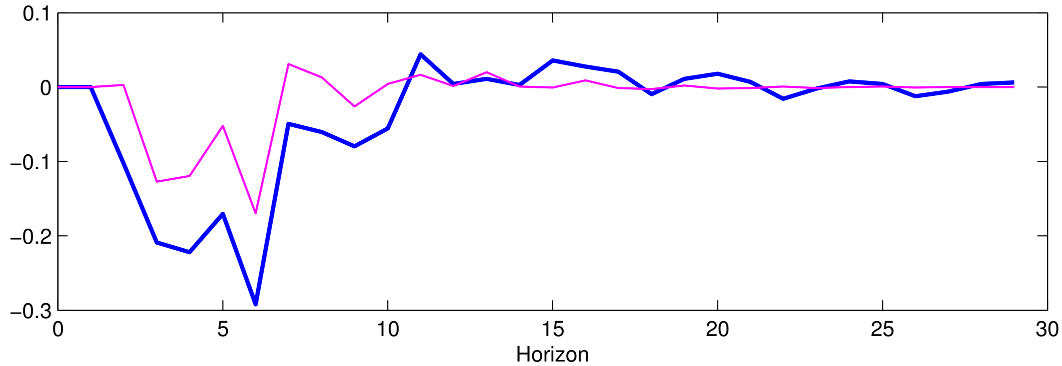

Figure 2: Comparison of the Impulse Response Functions

The thick line is the impulse response of the aggregate Solow residual to one unit interest rate innovation obtained from the 2-industry model. The thin line represents the impulse response function based on bivariate VAR(5) of the aggregate Solow residual and interest rate. The lag length is supported by LR criterion, AIC, and BIC.

for the industry i. The LR test indicates that the interest rate does not have any predictive power for non-manufacturing industry, which is a majority of the economy. Surprisingly, including the interest rate variable into the non-manufacturing equation does not help improve the log-likelihood value at all. The null hypothesis is accepted at the 5% level. On the other hand, the unobserved quarterly TFP for manufacturing industry is Granger-caused by the interest rate. The null hypothesis is rejected even at the 1% level of significance.

Given this result, we will further explore details in manufacturing industries. For this reason, we split the manufacturing industry into two groups, non-durable and durable manufacturing industries. Now a vector of unobserved quarterly TFP, $\Delta \boldsymbol{w}_t$, becomes a (3×1) vector. Coefficients matrices in the model should also be expanded accordingly.

Let $\Delta w_{2,t}$ denote the unobserved quarterly TFP for manufacturing industry, and also let $\Delta w_{(2,1),t}$ and $\Delta w_{(2,2),t}$ be the unobserved quarterly TFP for non-durable and durable industries, respectively. According to the discrete version of (4), we have

$$\Delta w_{2,t} = \gamma_1 \Delta w_{(2,1),t} + \gamma_2 \Delta w_{(2,2),t}, \tag{25}$$

Table 2: Likelihood Ratio Test Results for 2- and 3-industry Model

	LL Value	d.f.	LR Stat
2-industry Model			
Unrestricted	-454.27		
Restricted Non-manufacturing	-454.96	4	1.38
Restricted Manufacturing	-463.80	4	19.06**
3-industry Model			
Unrestricted	-497.78		
Restricted Non-durable	-508.54	4	21.52**
Restricted Durable	-508.47	4	21.38**

Note: The 5% level critical value for $\chi^2(4) = 9.488$. The 1% level critical value for $\chi^2(4) = 13.277$. ** indicates rejecting the null hypothesis of no Granger-causality at the 1% level.

where γ_1 and γ_2 are gross output share of non-durable and durable in manufacturing industry, respectively. Given the estimated parameter values in the 2-industry model, we can infer elements in $\mathbf{\Phi}_i$ that correspond to non-manufacturing and interest rate equations by using the relation in (25). Given the estimates for $\mathbf{\Phi}_j$ in the 2-industry model,

$$\hat{\mathbf{\Phi}}_{j,2\text{ind}} = \begin{bmatrix} \hat{\rho}_{11}^{(j)} & \hat{\rho}_{12}^{(j)} & \hat{g}_1^{(j)} \\ \hat{\rho}_{21}^{(j)} & \hat{\rho}_{22}^{(j)} & \hat{g}_2^{(j)} \\ \hat{h}_1^{(j)} & \hat{h}_2^{(j)} & \hat{f}^{(j)} \end{bmatrix},$$

we will get coefficient matrices $\mathbf{\Phi}_j$ for the 3-indusrty model as

$$\mathbf{\Phi}_{j,3\text{ind}} = \begin{bmatrix} \hat{\rho}_{11}^{(j)} & \gamma_1\hat{\rho}_{12}^{(j)} & \gamma_2\hat{\rho}_{12}^{(j)} & \hat{g}_1^{(j)} \\ \rho_{1,(2,1)}^{(j)} & \rho_{(2,1),(2,1)}^{(j)} & \rho_{(2,1),(2,2)}^{(j)} & g_{(2,1)}^{(j)} \\ \rho_{1,(2,2)}^{(j)} & \rho_{(2,2),(2,1)}^{(j)} & \rho_{(2,2),(2,2)}^{(j)} & g_{(2,2)}^{(j)} \\ \hat{h}_1^{(j)} & \gamma_1\hat{h}_2^{(j)} & \gamma_2\hat{h}_2^{(j)} & \hat{f}^{(j)} \end{bmatrix}. \tag{26}$$

Parameters in the shaded area in (26) are unknown. Since our purpose is to investigate the source of what happens at the higher level of aggregation with more disaggregated data, for comparability of results we just estimate those parameters in the shaded region by fixing other parameters at the estimates from the 2-industry model. This also helps us deal with

Table 3: Subcategories of Manufacturing Industries

Non-durable Manufacturing		Durable Manufacturing	
Consumer Products	Food and kindred products Tobacco Textile mill products Apparel Paper and allied Printing, publishing and allied Leather	Consumer Products	Furniture and fixtures Misc. manufacturing
		Materials	Lumber and wood Stone, clay, glass Primary metal Fabricated metal
Materials	Chemicals Petroleum and coal products Rubber and misc plastics	Processing and Assembly	Machinery, non-electrical Electrical machinery Motor vehicles Transp. equip. & ordnance Instruments

the increasing number of coefficients with more disaggregated data.

The lower panel of Table 2 presents the maximum log-likelihood values and the associated LR test statistics for the 3-industry model. The LR test results suggest that the interest rate does Granger-cause the industry-level quarterly TFP in both non-durable and durable manufacturing industries. The null hypothesis of no Granger-causality is rejected at the 1% level for both industries.

Now we will take a detailed look at both non-durable and durable manufacturing industries. To do so, we will categorize them into three subgroups based on characteristics of those individual industries. This step is necessary because there are many individual industries in both non-durable and durable industries (10 and 11 subindustries, respectively). If we move from non-durable or durable to individual industries, we will face the large number of parameters to be estimated at once. In order to manage such circumstances, we need to introduce an intermediate step.

Three subcategories in manufacturing industry are consumer products manufacturing, materials producing manufacturing, and processing and assembly manufacturing. The detailed classification of individual industries is shown in Table 3.

Based on the subcategories described above, we assume additional restrictions on coefficients to be estimated. These restrictions are about feedbacks from other groups of

Table 4: Wald Test Results for Subcategories in Manufacturing

Non-durable Manufacturing		Durable Manufacturing	
Consumer Products	8.536	Consumer Products	17.086^{**}
Materials	34.179^{**}	Materials	10.883^{*}
		Assembly & Processing	25.433^{**}

Note: Numbers reported are the Wald test statistics. The 5% level critical value for $\chi^2(4) = 9.488$. The 1% level critical value for $\chi^2(4) = 13.277$. ** indicates rejecting the null hypothesis of no Granger-causality at the 1% level.

industries. At this level of disaggregation, some groups of industries are irrelevant to other groups of industries in terms of production structure. If that is the case, TFP for other industries should not help predict TFP for a particular industry. For instance, none of the manufacturing industries uses tobacco products as material inputs, and thus technological progress in the tobacco industry should not help forecast TFPs in other industries. Such a priori restrictions are derived from the sparsity of the *use* table of the input-output accounts. If usage of outputs from a particular industry as intermediate inputs is less than 5% of total material inputs, we set the corresponding coefficient of feedback term to be zero.[10] These a priori restrictions on feedback terms help reduce the number of parameters to be estimated, especially at the very disaggregated level.

We will test non-durable and durable industries one by one. As before, we estimate parameters related to subcategories of interest, but leave other parameters fixed at the estimates from the less disaggregated model. Suppose, for example, we take subcategories of non-durable manufacturing. Then coefficients in the equations describing the behavior of TFP in non-manufacturing and durable manufacturing are fixed at the estimates from the 3-industry model, and vice versa.

Table 4 summarizes the Wald test results for subcategories of non-durable and durable industries. The Wald test for the null hypothesis of no Granger-causality suggests that we fail to reject the null only for non-durable consumer products manufacturing. Other

[10]These restrictions are similar to the idea of Shea (1993). He uses the sparsity of the IO table in order to search for instrument variables at disaggregated industries. We use 1977 benchmark IO table to determine the sparsity.

Table 5: Wald Test Results for Detailed Manufacturing Industries

Non-durable - Materials	
Chemicals	25.024**
Petroleum & Coal Products	85.801**
Rubber & Misc Plastics	20.451**
Durable - Consumer Products	
Furniture & Fixture	22.477**
Misc. Manufacturing	21.773**
Durable - Materials	
Lumber & Wood	25.853**
Stone, Clay, Glass	17.213**
Primary Metal	33.351**
Fabricated Metal	32.061**
Durable - Processing & Assembly	
Non-electrical Machinery	8.107
Electrical Machinery	4.320
Motor vehicles	13.830**
Transportation Equipment	20.837**
Instruments	13.587**

Note: Numbers reported are the Wald test statistics. The 5% level critical value for $\chi^2(4) = 9.488$. The 1% level critical value for $\chi^2(4) = 13.277$. ** indicates rejecting the null hypothesis of no Granger-causality at the 5% and 1% level.

subcategories of manufacturing industries continue to show the evidence of the Granger-causality. Except for materials producing durable industry, the null hypothesis is rejected at the 1% level.

Finally, we will turn to analyzing the Granger-causality found at the highly aggregated level with very disaggregated data. We take non-durable materials producing and three subcategories of durable manufacturing industries, and decompose each subindustry one by one. We will look at the model in which one of those subcategories is broken down into individual industries. For instance, if we focus on the non-durable material producing industries, industries included in the model will be non-manufacturing, non-durable consumer products producing, durable manufacturing, and those individual industries. As described above, we impose a priori restrictions on coefficients.

A summary of the Wald test results appears in Table 5. As we can see, the interest rate

Granger-causes the unobserved quarterly TFP for most of individual industries, except for both non-electric and electric machinery. The null hypothesis is rejected at the 1% level significance.

4 Extension

In this section, we will explore one of possibilities that interest rates can Granger-cause the aggregate Solow residual. Although several explanations have been offered, we will restrict our attention to a possibility of variable capital utilization. In Jorgenson's data set, the capital utilization rate is assumed to be constant. In the presence of variable capital utilization rate, instead of (3), the true technological progress can be expressed as:

$$\frac{dZ_i}{Z_I} = \frac{dQ_i}{Q_i} - \left\{ c^K \left(\frac{dK_i}{K_i} + \frac{dU_i}{U_i} \right) + c^L \frac{dL_i}{L_i} + c^E \frac{dE_i}{E_i} + c^M \frac{dM_i}{M_i} \right\},$$

where K_i represents the physical stock of capital, U_i is the rate of capital utilization, and $U_i K_i$ is assumed to provide service flow of capital. However, according to the growth accounting, we calculate technological progress as $\frac{dQ_i}{Q_i} - \left\{ c^K \frac{dK_i}{K_i} + c^L \frac{dL_i}{L_i} + c^E \frac{dE_i}{E_i} + c^M \frac{dM_i}{M_i} \right\}$. This means that the observed measure of technological progress is contaminated by growth rate of capital utilization.

Although the hypothesis of variable capital utilization is quite realistic, it is difficult to implement it into an empirical estimation. It is because we do not have an appropriate series for capital utilization. In general, the Federal Reserve's capacity utilization series are conceptually misleading as a proxy for capital utilization as argued by Shapiro (1989, 1996). So several researchers suggest to employ the use of materials (Basu, 1996) or the use of energy (Burnside, Eichenbaum, and Rebelo, 1995) as a proxy for capital utilization, by assuming low substitution between value added and those variables used as proxies. In stead, Shapiro (1996) advocates using the workweek of capital in order to proxy for capital utilization. Based on the Census Bureau's *Survey of Plant Capacity*, Beaulieu and Mattey (1998) construct the workweek of capital series that is publicly available. However, their data set covers only from 1974 to 1992 at an annual frequency.

Beaulieu and Mattey (1998) convincingly argue that for a group of industries that can be characterized as continuous processing industries, the FRB's measure of capacity utilization is likely to be better than the workweek of capital as a proxy for capital utilization.[11] The reason is that for those continuous processing industries, almost all of the variation in capital utilization arises from variation in the speed or intensity of capital use, not the duration of operations, which is captured by the workweek of capital.

Among those individual industries that appear to show the evidence of the Granger-causality in Section 3, chemicals and petroleum and coal products fall into the category of the continuous-processing industries and corresponding the FRB's capacity utilization series are available from 1948.[12] Thus, we will take these two industries and see whether taking account of unobserved capital utilization based on the FRB's capacity utilization eliminates the sign of the Granger-causality. Since we are only interested in either chemicals or petroleum and coal products, we combine other industries together. Thus the economy consist of chemicals (or petroleum products) and the rest of industries.

Although the capacity utilization series can serve as a proxy for capital utilization, there is another issue that may affect our estimation. As Shapiro (1989) argues, there might be a substantial error in the capacity utilization data that comes from interpolating the annual data on capacity. To cope with this issue, we assume that the capacity utilization series is subject to measurement error in log-level. That is,

$$u^*_{i,t} = u_{i,t} + m_{i,t}$$

where $u^*_{i,t}$ is the observed log-level of utilization of the industry i, $u_{i,t}$ is the true log-level of

[11] Their argument is based on the classification done by Mattey and Strongin (1997). The US four-digit manufacturing industries are categorized into three groups. *Variable work period group*, where plants such as automakers tend to vary output through adjusting the workweek; *Continuous-processors*, where manufactures vary their output by adjusting its speed of throughput; *Other manufactures*, where producers change their output rate through adjusting other inputs such as momentary labor input.

[12] Percentages of four-digit continuous-processing industries in chemicals and petroleum products are 80.6% and 93.7%, respectively.

utilization, and $m_{i,t}$ represents a white noise measurement error. This leads to an additional observation equation

$$\Delta u^*_{i,t} = \Delta u_{i,t} + m_{i,t} - m_{i,t-1}.$$

As stated earlier, the observed TFP is a linear combination of the true technological progress and the growth rate of unobserved capital utilization. Thus the observation equation for the industry-level TFP should be modified as follows:

$$\begin{aligned}\Delta z_{i,t/4} = \frac{1}{4}\{\Delta w_{i,t} + 2\Delta w_{i,t-1} + 3\Delta w_{i,t-2} + \\ 4\Delta w_{i,t-3} + 3\Delta w_{i,t-4} + 2\Delta w_{i,t-5} + \Delta w_{i,t-6}\} + \\ \frac{c^K}{4}\{\Delta u_{i,t} + 2\Delta u_{i,t-1} + 3\Delta u_{i,t-2} + \\ 4\Delta u_{i,t-3} + 3\Delta u_{i,t-4} + 2\Delta u_{i,t-5} + \Delta u_{i,t-6}\}.\end{aligned}$$

Now, instead of (13), the aggregate Solow residual can be represented as:

$$\Delta a_t = d_1 \Delta w_{1,t} + d_2(\Delta w_{2,t} + c^K \Delta u_{2,t}) + b + \nu_t.$$

where the subscript $i = 1$ represents the rest of industries and the subscript $i = 2$ corresponds to either chemicals or petroleum and coal products. Thus $\boldsymbol{y}_t$ contains Δa_t, Δx_t, $\Delta u^*_{2,t}$ and $\Delta \boldsymbol{z}_{t/4}$ in the fourth quarter and, as before, $\Delta \boldsymbol{z}_{t/4}$ does not enter $\boldsymbol{y}_t$ in other quarters. $\boldsymbol{k}_t$ now includes two additional components, $\Delta u_{2,t}$ and m_t. The remaining parts of the state space model are unchanged.

Table 6 presents the estimated results with the variable capital utilization for chemicals and petroleum and coal products.

Since we have an additional observed series, including the capital utilization into the system does not nest the previous specifications shown in Section 3. In order to see whether incorporating the capital utilization rate matters, first we compare the unrestricted model with the restricted model where we set all feedbacks from the capital utilization to the

Table 6: Estimation Results with Capital Utilization

		LL Value	d.f.	LR Stat
Chemicals				
Unrestricted		-640.759		
Restricted	$\boldsymbol{\alpha} = \boldsymbol{\beta} = \mathbf{0}$	-691.296	7	101.074**
	$\boldsymbol{g}_2 = \mathbf{0}$	-644.426	4	7.334
Petroleum and coal products				
Unrestricted		-604.059		
Restricted	$\boldsymbol{\alpha} = \boldsymbol{\beta} = \mathbf{0}$	-661.780	7	115.442**
	$\boldsymbol{g}_2 = \mathbf{0}$	-606.503	4	4.888

Note: $\boldsymbol{\alpha}$ is a vector of feedback coefficients from $\Delta u_{2,t}$ and $\boldsymbol{\beta}$ represents a vector of feedback coefficients from other variables to the capital utilization equation. $\boldsymbol{g}$ is a vector that collects coefficients on the interest rate for the chemical (or petroleum) equation. The 5% level critical values for $\chi^2(7) = 14.067$ and for $\chi^2(4) = 9.488$. The 1% level critical value for $\chi^2(7) = 18.475$ and $\chi^2(4) = 13.277$. ** indicates the rejection of the null hypothesis at the 1% level.

equations for other variables and all feedbacks from other variables to the equation for the capital utilization rate. That is, now the state-space model consists of two independent sets of equations, equations that inherit the previous specification and an equation of an AR process for the capital utilization rate. The first LR statistic in each panel of Table 6 shows strong evidences that the capital utilization is important to explain behaviors of unobserved TFP growth and the interest rate. For both industries, the null hypothesis of no effects is rejected at the 1% significance level.

Now we will look at whether the predictive power of the interest rate vanishes when we control for the unobserved capital utilization. The LR test results (the second LR statistic in each panel of Table 6) reveal that once we account for variable capital utilization with an appropriate measure, the quarterly unobserved TFP in chemicals and petroleum and coal products industries are no longer Granger-caused by the interest rate variable. We fail to reject the null hypothesis of no Granger-causality at the 5% level of significance. This is consistent with the findings from Paquet and Robidoux (2001). Although they work with aggregate data in the Canadian economy, they report that once capital utilization is taken into consideration, the capital-utilization-adjusted Solow residual is no longer Granger-cause by a set of monetary variables.[13]

It may sound obvious, but using an appropriate measure of capital utilization is important. We also experiment with growth rate of capacity utilization for manufacturing industry, whose correlation with the growth rate of workweek of capital is 0.49, according to Beaulieu and Mattey (1998). Incorporating inappropriate measure of capital utilization results in rejection of the null hypothesis of no Granger causality from the interest rate to TPF.

5 Conclusion

We have been analyzing the predictive power of the interest rate for the aggregate Solow residual. Although the industry-level data is only available at an annual frequency, by using the state space model and the Kalman filter, we are able to perform the Granger-causality test on the disaggregated industry TFP at a quarterly frequency. Our approach will avoid problems arising from either converting high frequency data into low frequency data or interpolating low frequency observations when variables of our interest are available at different frequencies.

In terms of the forecasting power of the interest rate on the industry-level TFP growth, heterogeneity exists among industries. Furthermore, in most cases, the predictive power of the interest rate, if any, is statistically very significant at the 1% level.

Our results suggest that manufacturing industries are responsible for the non-exogenous nature of the aggregate Solow residual reported by Evans (1992). From the 2-industry model, we have found that while the interest rate helps predict TFP growth in manufacturing industry, productivity growth in non-manufacturing industry is not Granger-caused by the interest rate. Although TFP growth in non-durable manufacturing industry as a whole appears to be Granger-caused by the interest rate, exploring the details reveals that the interest rate plays an important role only in three industries that characterized as ma-

[13] Paquet and Robidoux (2001) use the capacity utilization series published by the Statistics Canada to capture the effect of variable capital utilization. They claim that it is conceptually superior to counterparts in other countries.

terials producing industries, such as chemicals, petroleum and coal products, and rubber and misc plastics. Contrary to the findings in non-durable manufacturing, vast majority of durable manufacturing industries shows the evidence of the Granger causality, except for non-electrical machinery and electrical machinery.

We also extend our analysis by taking account of variable capital utilization. Two case studies on chemical and petroleum and coal products, for which we have an appropriate measure of capital utilization, show that once we take account of variable capital utilization, the TFPs in those industries are no longer Granger-caused by the interest rate. This result suggests that the predictive power of the interest rate will disappear if we control for variable capital utilization with an appropriate measure.

References

Barsky, R. B., and J. A. Miron (1989): "The Seasonal Cycle and the Business Cycle," *Journal of Political Economy*, 97(3), 503–534.

Basu, S. (1996): "Procyclical Productivity: Increasing Returns or Cyclical Utilization," *Quarterly Journal of Economics*, 111(3), 719–751.

Basu, S., and J. G. Fernald (1995): "Are Apparent Productive Spillovers a Figment of Specification Error?," *Journal of Monetary Economics*, 36(1), 165–188.

——— (2001): "Why is Productivity Procyclical? Why Do We Care?," in *New Developments in Productivity Analysis*, ed. by C. R. Hulten, E. R. Dean, and M. J. Harper, vol. 63 of *Studies in Income and Wealth*, chap. 7, pp. 225–301. University of Chicago Press.

Basu, S., J. G. Fernald, and M. S. Kimball (2004): "Are Technology Improvements Contractionary?," *NBER Working Paper*, No. 10592.

Basu, S., J. G. Fernald, and M. D. Shapiro (2001): "Productivity Growth in the

1990s: Technology, Utilization, or Adjustment?," *Carnegie-Rochester Conference Series on Public Policy*, 55, 117–165.

Basu, S., and M. S. Kimball (1997): "Cyclical Productivity with Unobserved Input Variation," *NBER Working Paper*, No. 5915.

Beaulieu, J. J., and J. Mattey (1998): "The Workweek of Capital and Capital Utilization in Manufacturing," *Journal of Productivity Analysis*, 10(2), 199–223.

Burmeister, E., K. D. Wall, and J. D. Hamilton (1986): "Estimation of Unobserved Expected Monthly Inflation Using Kalman Filter," *Journal of Business & Economic Statistics*, 4(2), 147–160.

Burnside, C., M. Eichenbaum, and S. Rebelo (1995): "Capital Utilization and Returns to Scale," in *NBER Macroeconomics Annual*, ed. by B. S. Bernanke, and J. J. Rotemberg, pp. 67–110. MIT Press.

——— (1996): "Sectoral Solow Residuals," *European Economic Review*, 40(3-5), 861–869.

Domar, E. D. (1961): "On the Measurement of Technological Change," *Economic Journal*, 71(284), 709–729.

Evans, C. L. (1992): "Productivity Shocks and Real Business Cycles," *Journal of Monetary Economics*, 29(2), 191–208.

Hamilton, J. D. (1985): "Uncovering Financial Market Expectations of Inflation," *Journal of Political Economy*, 93(6), 1224–1241.

——— (1994): *Time Series Analysis*. Princeton University Press, Princeton.

Harvey, A. C. (1989): *Forecasting, structural time series models and the Kalman filter*. Cambridge University Press, Cambridge.

Harvey, A. C., and R. G. Pierse (1984): "Estimating Missing Observations in Economic Time Series," *Journal of the American Statistical Association*, 79(385), 125–131.

Hulten, C. R. (2001): "Total Factor Productivity: A Short Biography," in *New Developments in Productivity Analysis*, ed. by C. R. Hulten, E. R. Dean, and M. J. Harper, vol. 63 of *Studies in Income and Wealth*, chap. 1, pp. 1–47. University of Chicago Press.

Ireland, P. N. (2004): "A Method for Taking Models to the Data," *Journal of Economic Dynamics & Control*, 28(6), 1205–1226.

Jorgenson, D. W. (1990): "Productivity and Economic Growth," in *Fifty Years of Economic Measurement*, ed. by E. R. Berndt, and J. E. Triplett, vol. 54 of *Studies in Income and Wealth*, chap. 3, pp. 19–118. University of Chicago Press.

Jorgenson, D. W., F. M. Gollap, and B. M. Fraumeni (1987): *Productivity and U.S. Economic Growth*. Harvard University Press, Cambridge.

Jorgenson, D. W., and K. J. Stiroh (2000): "Raising the Speed Limit: U.S. Economic Growth in the Information Age," *Brookings Papers on Economic Activity*, (1), 125–211.

Mattey, J., and S. Strongin (1997): "Factor Utilization and Margins for Adjusting Output: Evidence from Manufacturing Plants," *Federal Reserve Bank of San Francisco Economic Review*, (2), 3–17.

Paquet, A., and B. Robidoux (2001): "Issues on the Measurement of the Solow Residual and the Testing of its exogeneity: Evidence for Canada," *Journal of Monetary Economics*, 47(3), 595–612.

Sato, K. (1975): *Production Functions and Aggregation*. North-Holland, Amsterdam.

——— (1976): "The Meaning and Measurement of the Real Value Added Index," *Review of Economics and Statistics*, 58(4), 434–442.

Shapiro, M. D. (1989): "Assessing the Federal Reserve's Measures of Capacity and Utilization," *Brookings Papers on Economic Activity*, (1), 181–225.

——— (1996): "Macroeconomic Implications of Variation in the Workweek of Capital," *Brookings Papers on Economic Activity*, (2), 79–119.

Shea, J. (1993): "The Input-Output Approach to Instrument Selection," *Journal of Business & Economic Statistics*, 11(2), 145–155.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1(5), 1123–1175.

Appendix

A The Kalman filter and maximum likelihood estimation

Given $\hat{\boldsymbol{\xi}}_{1|0}$, $\boldsymbol{P}_{1|0}$, and parameter values, the Kalman filter recursively calculate $\{\hat{\boldsymbol{\xi}}_{t+1|t}\}_{t=1}^{T}$ and $\{\boldsymbol{P}_{t+1|t}\}_{t=1}^{T}$ as:

$$\begin{aligned}
\hat{\boldsymbol{\xi}}_{t+1|t} &= \boldsymbol{\Phi}\hat{\boldsymbol{\xi}}_{t|t-1} + \boldsymbol{\Phi}\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}_t'(\boldsymbol{\Psi}_t\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}_t' + \boldsymbol{R}_t)^{-1}(\boldsymbol{y}_t - \boldsymbol{\Psi}_t\hat{\boldsymbol{\xi}}_{t|t-1} - \boldsymbol{b}_t), \\
\boldsymbol{P}_{t+1|t} &= \boldsymbol{\Phi}\boldsymbol{P}_{t|t-1}\boldsymbol{\Phi}' - \boldsymbol{\Phi}\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}_t'(\boldsymbol{\Psi}_t\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}_t' + \boldsymbol{R}_t)^{-1}\boldsymbol{\Psi}_t\boldsymbol{\Phi}\boldsymbol{P}_{t|t-1}\boldsymbol{\Phi}' + \boldsymbol{Q}.
\end{aligned}$$

The log likelihood function is given by

$$L = \sum_{t=1}^{T}\left\{-\frac{n_t}{2}\log(2\pi) - \frac{1}{2}\log\left(|\boldsymbol{\Omega}_t|\right) + \boldsymbol{e}_t'\boldsymbol{\Omega}_t^{-1}\boldsymbol{e}_t\right\},$$

where n_t is dimension of $\boldsymbol{y}_t$,

$$\begin{aligned}
\boldsymbol{e}_t &= \boldsymbol{y}_t - \boldsymbol{\Psi}_t\hat{\boldsymbol{\xi}}_{t|t-1} - \boldsymbol{b}_t, \text{and} \\
\boldsymbol{\Omega}_t &= \boldsymbol{\Psi}_t\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}_t' + \boldsymbol{R}_t.
\end{aligned}$$

For the estimation, we set $\hat{\boldsymbol{\xi}}_{1|0}$ and $\boldsymbol{P}_{1|0}$ as follows:

$$\hat{\boldsymbol{\xi}}_{1|0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad \boldsymbol{P}_{1|0} = 20 \times \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0}' & \cdots & \cdots & \boldsymbol{0}' & 0 \end{bmatrix}$$

B Gradient and Information Matrix

The derivative of the log-likelihood function at t with respect to the i^{th} parameter is given by

$$\frac{\partial \log L_t}{\partial \theta_i} = -\frac{1}{2}\text{tr}\left\{ \left(\boldsymbol{\Omega}_t^{-1} \frac{\partial \boldsymbol{\Omega}_t}{\partial \theta_i} \right) \left(\boldsymbol{I} - \boldsymbol{\Omega}_t^{-1} \boldsymbol{e}_t \boldsymbol{e}_t' \right) \right\} - \left(\frac{\partial \boldsymbol{e}_t}{\partial \theta_i} \right)' \boldsymbol{\Omega}_t^{-1} \boldsymbol{e}_t.$$

Derivatives of $\boldsymbol{e}_t$ and $\boldsymbol{\Omega}_t$ are

$$\frac{\partial \boldsymbol{e}_t}{\partial \theta_i} = -\boldsymbol{\Psi}_t \frac{\partial \hat{\boldsymbol{\xi}}_{t|t-1}}{\partial \theta_i} - \frac{\partial \boldsymbol{b}_t}{\partial \theta_i}, \quad (27)$$

$$\frac{\partial \boldsymbol{\Omega}_t}{\partial \theta_i} = \boldsymbol{\Psi}_t \frac{\partial \boldsymbol{P}_{t|t-1}}{\partial \theta_i} \boldsymbol{\Psi}_t' + \frac{\partial \boldsymbol{R}_t}{\partial \theta_i}, \quad (28)$$

respectively.

Let us define $\boldsymbol{M}_t = \Phi \boldsymbol{P}_{t|t-1} \Psi_t'$. Then the Kalman recursion can be expressed as:

$$\hat{\boldsymbol{\xi}}_{t+1|t} = \Phi \hat{\boldsymbol{\xi}}_{t|t-1} + \boldsymbol{M}_t \boldsymbol{\Omega}_t^{-1} \boldsymbol{e}_t,$$

$$\boldsymbol{P}_{t+1|t} = \Phi \boldsymbol{P}_{t|t-1} \Phi' - \boldsymbol{M}_t \boldsymbol{\Omega}_t^{-1} \boldsymbol{M}_t' + \boldsymbol{Q}.$$

Given (27), (28), and

$$\frac{\partial \boldsymbol{M}}{\partial \theta_i} = \frac{\partial \boldsymbol{\Phi}}{\partial \theta_i} \boldsymbol{P}_{t|t-1} \boldsymbol{\Psi}_t' + \boldsymbol{\Phi} \frac{\partial \boldsymbol{P}_{t|t-1}}{\partial \theta_i} \boldsymbol{\Psi}_t',$$

we can calculate

$$\frac{\partial \hat{\boldsymbol{\xi}}_{t+1|t}}{\partial \theta_i} = \frac{\partial \Phi}{\partial \theta_i} \hat{\boldsymbol{\xi}}_{t|t-1} + \Phi \frac{\partial \hat{\boldsymbol{\xi}}_{t|t-1}}{\partial \theta_i} + \frac{\partial \boldsymbol{M}_t}{\partial \theta_i} \boldsymbol{\Omega}_t^{-1} \boldsymbol{e}_t - \boldsymbol{M}_t \boldsymbol{\Omega}_t^{-1} \frac{\partial \boldsymbol{\Omega}_t}{\partial \theta_i} \boldsymbol{\Omega}_t^{-1} \boldsymbol{e}_t + \boldsymbol{M}_t \boldsymbol{\Omega}_t^{-1} \frac{\partial \boldsymbol{e}_t}{\partial \theta_i},$$

$$\frac{\partial \boldsymbol{P}_{t+1|t}}{\partial \theta_i} = \frac{\partial \Phi}{\partial \theta_i} \boldsymbol{P}_{t|t-1} \Phi' - \frac{\partial \boldsymbol{M}_t}{\partial \theta_i} \boldsymbol{\Omega}_t^{-1} \boldsymbol{M}_t' + \boldsymbol{M}_t \boldsymbol{\Omega}_t^{-1} \frac{\partial \boldsymbol{\Omega}_t}{\partial \theta_i} \boldsymbol{\Omega}_t^{-1} \boldsymbol{M}_t' - \boldsymbol{M}_t \boldsymbol{\Omega}_t^{-1} \frac{\partial \boldsymbol{M}_t'}{\partial \theta_i} + \frac{\partial \boldsymbol{Q}}{\partial \theta_i}.$$

By going through the Kalman recursion, we can calculate $\{\frac{\partial \hat{\boldsymbol{\xi}}_{t+1|t}}{\partial \boldsymbol{\theta}}\}_{t=1}^T$ and $\{\frac{\partial \hat{\boldsymbol{\xi}}_{t+1|t}}{\partial \boldsymbol{\theta}}\}_{t=1}^T$. Once we calculate $\frac{\partial \boldsymbol{e}_t}{\partial \theta_i}$ and $\frac{\partial \boldsymbol{\Omega}_t}{\partial \theta_i}$, we can easily compute the Information matrix. The (i, j)

element of the information matrix is given by

$$\boldsymbol{I}_{ij}(\boldsymbol{\theta}) = \frac{1}{2}\sum_{t=1}^{T}\left\{\text{tr}\left(\boldsymbol{\Omega}_t^{-1}\frac{\partial\boldsymbol{\Omega}_t}{\partial\theta_i}\boldsymbol{\Omega}_t^{-1}\frac{\partial\boldsymbol{\Omega}_t}{\partial\theta_j}\right)\right\} + \sum_{t=1}^{T}\left(\frac{\partial\boldsymbol{e}_t}{\partial\theta_i}\right)'\boldsymbol{\Omega}_t^{-1}\frac{\partial\boldsymbol{e}_t}{\partial\theta_j}.$$

C A Monte Carlo Experiment

In this section, we consider the simplified version of the state-space model used in the paper. It corresponds to combining quarterly and semi-annual series. With generated observations, we perform the Granger-causality test and see whether we can get correct inference or not.

State Equation: A vector of unobservables is assumed to evolve as

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} 0.7 & 0.5 & -0.3 \\ \phi_{21} & 0.6 & 0.5 \\ \phi_{31} & -0.2 & 0.4 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} + \begin{bmatrix} \varepsilon^x_{t+1} \\ \varepsilon^y_{t+1} \\ \varepsilon^z_{t+1} \end{bmatrix} \tag{29}$$

where $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \boldsymbol{I}$.

Observation Equation: For $t = 1, 3, 5, \cdots,$

$$\begin{bmatrix} a_t \\ x_t \end{bmatrix} = \begin{bmatrix} 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{\xi}_t + \begin{bmatrix} \nu_t \\ 0 \end{bmatrix} \tag{30}$$

For $t = 2, 4, 6, \cdots,$

$$\begin{bmatrix} a_t \\ x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 1 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 1 & 0 & 0 & 0.5 & 0 \end{bmatrix} \boldsymbol{\xi}_t + \begin{bmatrix} \nu_t \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{31}$$

where $\boldsymbol{\xi}_t = [\ x_t \quad y_t \quad z_t \quad x_{t-1} \quad y_{t-1} \quad z_{t-1} \quad x_{t-2} \quad y_{t-2} \quad z_{t-2} \quad 1\ ]'$ and $E[\nu_t^2] = 1$.

We set $\phi_{21} = \phi_{31} = 0$. That is, x_t does not Granger-cause y_t, nor z_t. Generate observables using the above system and estimate parameters. Then we calculate the Wald statistic for the null hypothesis $\phi_{21} = \phi_{31} = 0$ and repeat 1000 times. Results from Monte Carlo 1000 replications are shown in Table 7.

Table 7: Monte Carlo Results with 1000 Replications

Sample Size	Prob($W > c_{0.05}(2)$)
T = 100	0.069
T = 200	0.053
T = 300	0.048

Note: $c_{0.05}(2)$ denotes the critical value of χ^2 distribution with d.f. = 2 at the 5% significance level. The second column reports a probability of Type I errors.